Probability of Two Events

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# Probability of Two Events

Let us recall from our previous lesson the following concepts on probability.

1. The probability of an event (E) is given by the formula:

𝑷(𝑬) = 𝒏𝒖𝒎𝒃𝒆𝒓 𝒐𝒇 𝒇𝒂𝒗𝒐𝒓𝒂𝒃𝒍𝒆 𝒐𝒖𝒕𝒄𝒐𝒎𝒆𝒔 𝒏(𝑬) / 𝒏𝒖𝒎𝒃𝒆𝒓 𝒐𝒇 𝒂𝒍𝒍 𝒑𝒐𝒔𝒔𝒊𝒃𝒍𝒆 𝒐𝒖𝒕𝒄𝒐𝒎𝒆𝒔 𝒏(𝑺)

1. The probability of an event must be a number from 0 to 1. That is, 𝟎 ≤ 𝑷(𝑬) ≤ 𝟏.
2. The union of events A and B, denoted by A ∪ B, is the set of all outcomes for either A or B.
3. The intersection of events A and B, denoted by A ∩ B, is the set of all outcomes shared by A and B.

### Exercise 1

Direction: Identify whether the events can happen at the same time or if they can’t happen at the same time. Write your answer at the column where they are belonged.

Events:

1. A = tossing a coin and getting a head

* B = tossing a coin and getting a tail

1. A = rolling a die and getting a factor of 6

* B = rolling a die and getting a prime number

1. A = a heart is drawn from a standard deck of cards

* B = a face card is drawn from a standard deck of cards

1. A = an ‘8’ is drawn from a standard deck of cards

* B = a king is drawn from a standard deck of cards

1. A = a multiple of 3 turning up in rolling a die once

* B = a factor of 4 turning up in rolling a die once

|  |  |
| --- | --- |
| It can happen at the same time | It can’t happen at the same time |
|  |  |

Were you able to identify the events that can occur at the same time and the events that cannot occur at the same time?

The pair of events in Exercise 1 are compound events and they could either be mutually exclusive or not mutually exclusive events.

## Compound Event

A compound event consists of two or more simple events that are connected by the word **and** or **or**.

## Mutually Exclusive Event

Two events are mutually exclusive if both events cannot occur at the same time. These events have no common elements. They are also called disjoint events.

Example 1. In tossing a coin once, the events of getting a head and getting a tail are mutually exclusive events because they cannot appear at the same time. If A is the event of getting a head and B is the event of getting a tail, then, 𝑨 ∩ 𝑩 = { } or 𝑛(𝑨 ∩ 𝑩) = 𝟎. That is, there is no common element in events A and B.

## Not Mutually Exclusive Event

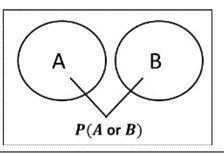
Two events are not mutually exclusive if both events can occur at the same time. These events have common elements. They are also called inclusive events.

Example 2. In rolling a six-sided die once, the events ‘a 2 turning up’ and ‘an even number turning up’ are not mutually exclusive events. If A is the event of ‘a 2 turning up’ and B is the event of ‘an even number turning up’, then, A = {2} and B = {2, 4, 6}. Observe that the number 2 is common to both events A and B, therefore, 𝑨 ∩ 𝑩 = {𝟐} 𝑜𝑟 𝑛(𝑨 ∩ 𝑩) = 𝟏.

## Probability of Mutually Exclusive Events

If two events, A and B, are mutually exclusive, then the probability that either A or B occurs is the sum of their probabilities. In symbols

𝑷(𝑨 𝐨𝐫 𝑩) = 𝑷(𝑨 ∪ 𝑩) = 𝑷(𝑨) + 𝑷(𝑩)



Example. A bag contains 4 blue marbles, 8 green marbles and 6 red marbles. Carlo draws one ball at random. What is the probability that the marble is either red or green?

Solution: A ball cannot be both red and green, so these are mutually exclusive events.

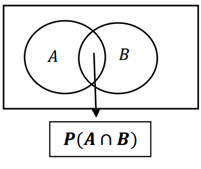
𝑃(red or green) = 𝑃(red) + 𝑃(green) = 6/18 + 8/18 = 14/18 or 7/9

The probability that the drawn marble is either red or green is 7/9.

## Probability of Not Mutually Exclusive Events

Probability of Not Mutually Exclusive Events For any two events A and B of the same experiment which are not mutually exclusive events, the probability of the union of A and B is

𝑷(𝑨 𝒐𝒓 𝑩) = 𝑷(𝑨) + 𝑷(𝑩) − 𝑷(𝑨 𝒂𝒏𝒅 𝑩) 𝑷(𝑨 ∪ 𝑩) = 𝑷(𝑨) + 𝑷(𝑩) − 𝑷(𝑨 ∩ 𝑩)



Example. A die is rolled once. What is the probability of an even number or a factor of 6 turning up?

Solution: A die has six faces numbered 1 – 6, so, 𝑛(𝑆) = 6. Let event: A = {even number} and B = {factor of 6}

A = {2, 4, 6} B = {1, 2, 3, 6} 𝐴 ∩ 𝐵 = {2, 6}

Now, let’s use the formula

𝑃(𝐴 ∪ 𝐵) = 𝑃(𝐴) + 𝑃(𝐵) − 𝑃(𝐴 ∩ 𝐵) = 3/6 + 4/6 - 2/6 = 5/6

The probability of getting an even number or a factor of 6 is 5/6.

### Activity 1

Direction: Write **ME** if the events are mutually exclusive and **NME** if it is not mutually exclusive.

1. Drawing ‘a jack’ and ‘a club’ from a standard deck of cards.
2. Drawing ‘a 7’ and ‘a 4’ from a standard deck of cards.
3. Picking ‘a blue ball’ and ‘a red ball’ in a basket.
4. Electing ‘the president’ and ‘the secretary’ of the class.
5. Getting ‘an even number’ and ‘a factor of 4’ in rolling a fair die once.

### Activity 2

Direction: Solve for its probability of two events. Show your complete solutions.

1. A card is drawn at random from a standard deck of 52 cards. What is the probability of drawing an ace, a 10 or a king?
2. In this ‘new normal situation’, nobody can go out without wearing a face mask. Mark has 15 disposable face masks: 4 are red, 6 are blue and 5 are green. What is the probability that he will wear a red or a blue face mask today?
3. If there is 30% chance of rain on Saturday, 70% chance of rain on Sunday, and 21% chance of rain on Saturday and Sunday, what is the probability that it will rain on either Saturday or Sunday?
4. Each of the numbers 1 – 30 is written on a slip of paper, rolled and put in a box and mixed thoroughly. One number is picked up at random. Find the probability that the picked number is even or a multiple of 5?
5. A box contains 2 red, 4 green, 5 blue and 3 yellow marbles. If a single random marble is chosen from the box, what is the probability that it is red or green marble?
6. In a math class of 30 students, 17 are boys and 13 are girls. On a unit test, 4 boys and 5 girls made an A grade. If a student is chosen at random from the class, what is the probability of choosing a girl or an A student?
7. A pair of dice is rolled. What is the probability that the two dice show the same number or that the sum of the numbers is less than 5?
8. If a card is drawn at random from a 52-deck of cards. Find the probability of getting a heart or a diamond.

### Assessment

Direction: Identify whether the events are mutually exclusive and not mutually exclusive. Solve for its probability. Show your complete solutions.

1. There are 12 DVDs, 7 video games, 14 CDs, and 3 videotapes on Jaime’s bedroom shelf. If Jaime selects an item at random from the shelf, what is the probability that it is a DVD or a video tape?
2. Karen’s book bag contains 3 novels, 1 biography, and 1 science book. Manny’s book bag contains 1 math book, 2 science books, and 1 poetry book. Each student selects a book at random from his or her bag. What is the probability that either Karen’s book is a novel or Manny’s book is a math book?
3. Sigmund picked a pair of socks out of his drawer without looking in the drawer. Sigmund has 3 pairs of black dress socks, 4 pairs of brown dress socks, 3 pairs of black sport socks, and 5 pairs of white sport socks. What is the probability that Sigmund will select a pair of black socks or a pair of sport socks?
4. There are 8 girls and 12 boys in Miss Reading’s homeroom. Five of the girls play sports and 3 do not play sports. Eight of the boys play sports and 4 do not play sports. If a student is selected at random, what is the probability that the student is a boy or plays sports? Express your answer as a fraction.